Contribution to the Design of RNS Architecture

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Motivation

Context

Elliptic Curve Cryptography or RSA using RNS Architecture

Goal

- Construct a basis producing the largest product of moduli
- Improving frequency of RNS Cox-Rower architecture by removing the final subtraction

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IPsec

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- VPN (ECDH) : 4 scalar multiplications

Cryptoequipment with 10000 VPN $\Leftrightarrow$ 90000 scalar multiplications
20 ms for scalar multiplication $\Leftrightarrow$ 30 minutes to boot all the VPNs...

Outline

- RNS & Cox-Rower Architecture
- Self-correcting k
- Moduli selection
- Implementation & Results
- Conclusion

RNS : Motivation

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Modular arithmetic computations over large numbers

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Classical solution

Multiple-precision algorithms (Handbook of Applied Crypto, GMP, ...)

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Classical solution

Multiple-precision algorithms (Handbook of Applied Crypto, GMP, ...)

multiple-precision → carry propagations

For hardware, this means :

- bad critical path,
- no straightforward parallelism

RNS : formally

Let $M = \prod_{i=1}^{n} m_i$ and $x_i = X \mod m_i$.

$$\begin{aligned} \varphi : \mathbb{Z}/M\mathbb{Z} &\rightarrow \mathbb{Z}/m_1\mathbb{Z} \times \cdots \times \mathbb{Z}/m_n\mathbb{Z} \\ X &\mapsto (x_1, x_2, \ldots, x_n) \end{aligned}$$

Chinese Remainder Theorem

$$\forall 1 \leqslant i < j \leqslant n, \gcd(m_i, m_j) = 1 \Leftrightarrow \varphi \text{ is an isomorphism}$$

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Reconstruction

$$X = \sum_i (x_i M_i^{-1} \mod m_i) M_i \mod M,$$

where $M_i = M/m_i$.

RNS : numbers modulo M using base $\mathcal{B} = \{m_1, m_2, \ldots, m_n\}$.

Montgomery & RNS : Base Extension

Montgomery Reduction

$$X \mod p \equiv \frac{X+(X\cdot(-p^{-1}) \mod M)p}{M}$$

1. $Q_M = X \cdot (-p^{-1}) \mod M$
2. $Q_{M'} = BE(Q_M, \mathcal{B}, \mathcal{B}')$
3. $S_{M'} = (X_{M'} + Q_{M'} \cdot p) \cdot M^{-1}$
4. $S_M = BE(S_{M'}, \mathcal{B}', \mathcal{B})$

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Base Extension

$\{x'_1, \ldots, x'_n\} = BE(X, \mathcal{B}, \mathcal{B}')$

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where $\xi(x_i) = x_i M_i^{-1} \mod m_i$.

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$$x'_j = \sum_i \xi(x_i) M_i - k(\xi(x_1), \ldots, \xi(x_n)) M \mod m'_j$$

Why using RNS ?

Advantages

- no carry propagation,
- enable parallelism,
- fast computation ($O(n)$ complexity in RNS vs $O(n^{\log_2(3)})$ in multiprecision for multiplications when using Karatsuba).

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Drawbacks

None

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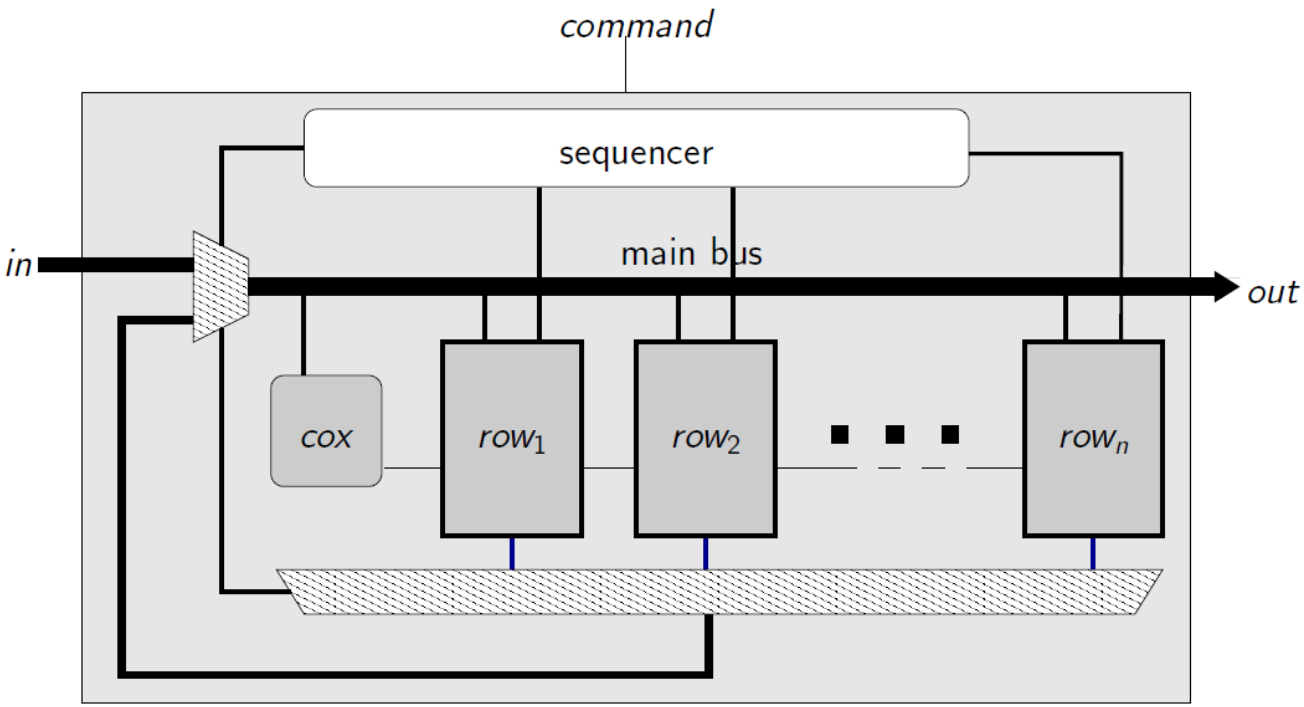
Drawbacks

- reduction over prime p is complex ($O(n^2)$ complexity in RNS),
- comparison is difficult,
- division modulo p is only possible for co-primes with M.

Cox-Rower Architecture

Features

- SIMD,
- Generic (ECC independent),
- Pipelinable,
- Suitable for lazy reduction ($\sum_{i=1}^{n} A_i B_i$).



Self-correcting k

Inner reduction without final subtraction

$\{x'_1, \ldots, x'_n\} = BE(X, \mathcal{B}, \mathcal{B}')$

$$X = \sum_i \xi_i M_i \mod M,$$

where $\xi_i \equiv x_i M_i^{-1} \mod m_i < 2m_i$.

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Proof (works also for Cox-Rower) :

$$\text{Let } \xi'_i = \xi_i + \beta_i m_i \text{ with } \beta_i = \{0,1\} \text{ and } k' = \left\lfloor \sum_{i=1}^{n} \frac{\xi'_i}{m_i} \right\rfloor = k + \sum_{i=1}^{n} \beta_i$$

$$\text{Then } X' = \sum_{i=1}^{n} \xi'_i M_i - k'M = \sum_{i=1}^{n} (\xi_i M_i + \beta_i M) - kM - \sum_{i=1}^{n} \beta_i M$$

Self-correcting k

Cox-Rower impact

Condition on moduli selection was : $2^r - \alpha 2^r \leqslant m_i \leqslant 2^r$,
New condition on moduli selection : $2^r - \alpha 2^{r-1} \leqslant m_i \leqslant 2^r$.

- No need to test for final subtract with m_i
- No need to modify the truncation ($\hat{k}$ evaluation) : error induced is cancelled

Moduli selection

Context

Cox-Rower RNS needs a set of coprime moduli in a certain range given a radix size

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How to find a maximal cardinality basis in given range ?

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Problems

How to find a maximal cardinality basis in given range ?

First come, first selected strategy

- Add 2^r, $2^r - 1$ since coprime, $2^r - 3$ if coprime, and so on...
- Can be very bad : $2^{16} - 1 = 65535 = 3 \times 5 \times 17 \times 257$

Goal : do not consume small prime factors too fast.

Moduli selection

Proposed strategy

Examine factorization of all numbers in the range :

- Doable in practice : range has 2^{16} for $r = 32$ bits
- Consider the small prime factors
- Can add numbers with exactly one directly :
  - Consumes exactly one small prime
  - Big primes appear only once

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Prime factors size

Prime factor size in the range $[\![2^r - \mu_{max}, 2^r]\!]$:

- Any prime greater than μ_{max} appear at most once as a factor in the range
- Especially true when $\mu_{max} = \sqrt{2^r}$ (each number has at most one prime factor greater than its square root)

Moduli selection

Proposed strategy : algorithms

Consume the least possible small primes :

- First add prime number in the range
- Add compatible numbers with exactly 1 small prime factor, greatest first :
  - There always exists a maximal basis containing these numbers
  - Remove non-coprime others candidates
  - Increase the big prime bound accordingly (as for 2^r where it is increased from μ_{max} to $2\mu_{max}$
- Compute a basis from the remaining numbers (using MAX-Clique solvers for example)

Moduli selection

In practice

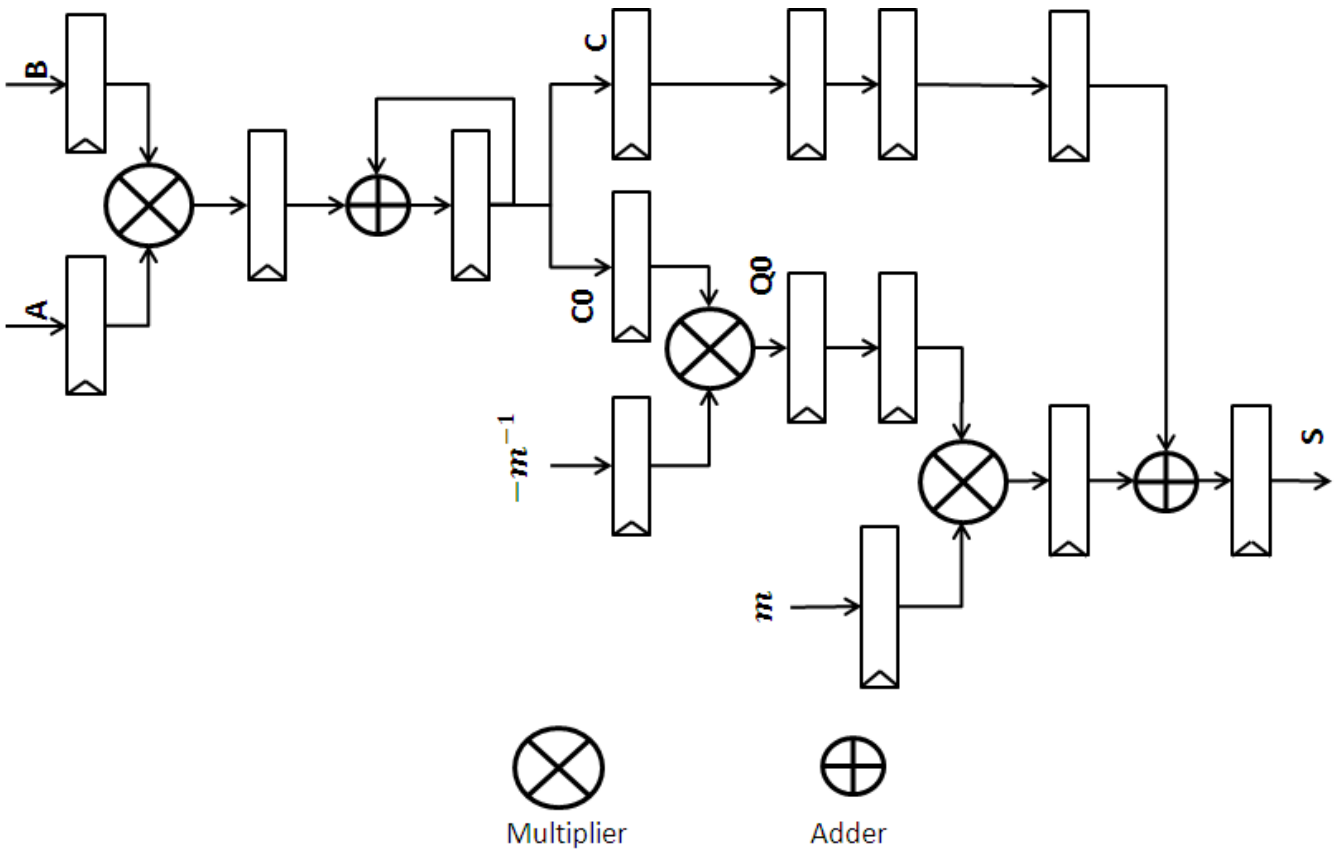
$r = 16$ bits, $\mu_{max} = 2^{r/2}$:

- First come first selected strategy : 43 elements, curve order security max about 165.5 bits,
- Proposed strategy : 48 elements, curve security max 189.5 bits,
- 24 bits security improvement, very short of attaining 192-bits security level

Implementation

Cox-Rower ALU

- No final reduction
- Adding pipeline and lazy reduction to improve frequency
- For Xilinx FPGA : fits entirely in DSP blocks



Results

Frequency limitation

In the sequencer block which decodes and translates instructions

Curve	n	Cycles	Slices (DSP/BRAM)	Fmax	Latency
160	11	88536	1168 (45/13)	400	0,221 ms
192	13	117732	1316 (53/15)	400	0,297 ms
224	15	150795	1542 (61/17)	400	0,377 ms
256	17	187828	1658 (69/19)	400	0,472 ms
384	25	373946	2630 (101/27)	400	0,935 ms
512	33	622053	3405 (133/33)	400	1,555 ms
521	33	632588	3435 (133/33)	400	1,581 ms

TABLE: P&R performances (Xilinx Kintex-7)

Results

Attainable security

Moduli range is large enough with inner Montgomery without final reduction with 16-bits multiplier

	With reduction		Without reduction
Mult. bitsize	Kawamura *et al.*	Inner Montgomery	Inner Montgomery
15	266	536	375
16	380	782	536
17	506	1118	782

TABLE: Attainable security for ALUs

Conclusion & Future works

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- Algorithm to find the largest product of coprime moduli given a range
- Critical path moved from datapath to sequencer

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- Architecture optimization for pairing computation
- Scalability to GPGPU

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Finally

Now, our cryptoequipment boots in 3 minutes !

Thank You !

Questions ?

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